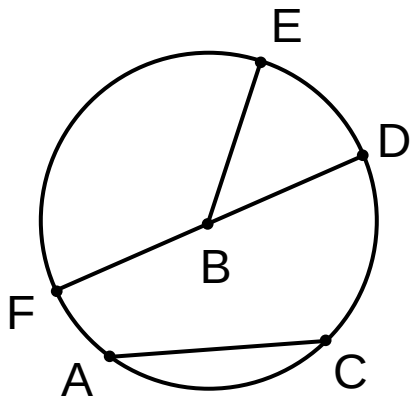
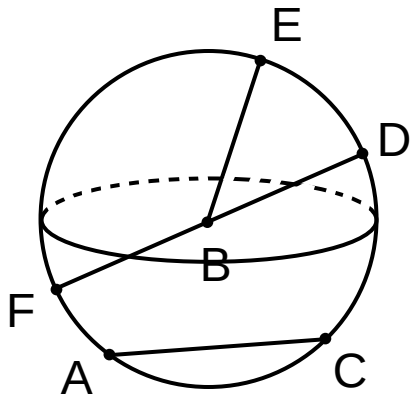
section 10.6

Definitions:

circle – the set of all points that are given distance from a given point in a given plane

sphere – the set of all points that are given distance from a given point in space





center – the given point in the plane (space)

radius of a circle – a segment that connects a point on the circle (sphere) with the center

chord of a circle – a segment having both endpoints on the circle (sphere)

diameter – a chord that passes through the center of a circle (sphere)

arc – a curve that is a subset of a circle (sphere)

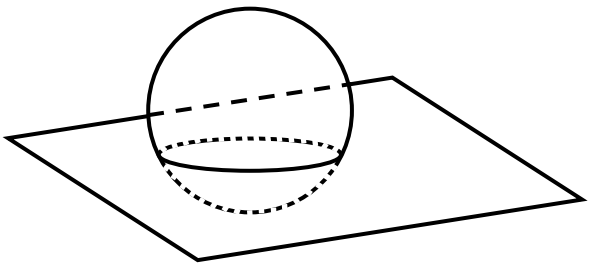
secant – a line (plane) that contains a chord

tangent – a line (plane) that contains exactly one point of the circle (sphere)

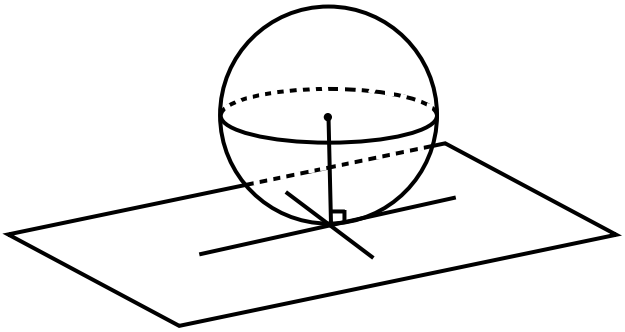
Intersections of Planes and Secants

Definitions:

A secant plane to a sphere is a plane that intersects a sphere in more than one point.



A tangent plane to a sphere is a plane that intersects a sphere in exactly one point. The point is called the point of tangency.



Theorem 10.19: The intersection of a sphere and a secant is a circle.

Definition:

A great circle of a sphere is the intersection of the sphere and a secant plane that contains the center of the sphere.

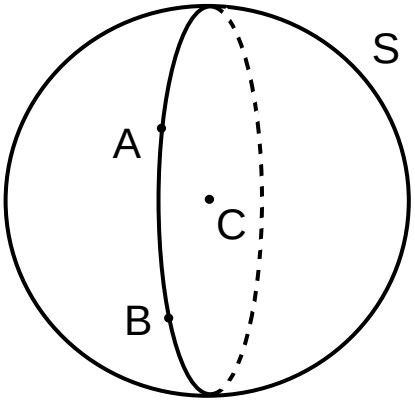
Note: The secant that contains the great circle splits the sphere into 2 equal halves (called hemispheres).

Theorem 10.20: Two points on a sphere that are not on the same diameter lie on exactly one great circle of the sphere.

Proof:

***Given*:** Sphere S with center C contains Points A and B that are not on the same diameter.

***Prove*:** A and B lie on exactly one great circle.



Statement	Reason
1. Sphere S with center C contains Points A and B that are not on the same diameter.	1. Given
2. A, B, C are not collinear	2. Def. of collinear (If they were collinear, then they would be on a diameter)
3. A, B, C determine exactly one plane p	3. Plane Postulate
4. p ∩ s is a great circle	4. Definition of great circle

Theorem 10.21: Two great circles of a sphere intersect at two points that are endpoints of a diameter of the sphere.

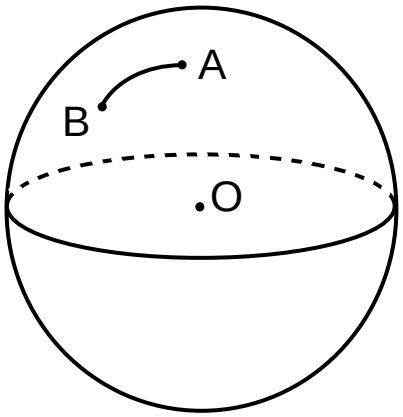
Theorem 10.22: All great circles of a sphere are congruent.

Theorem 10.23: A secant plane of a sphere is perpendicular to the line containing the center of the circle of intersection and the center of the sphere.

Theorem 10.24: A plane is tangent to a sphere if and only if it is perpendicular to the radius at the point of tangency.

Note: This is similar to a tangent line being perpendicular to a radius of a circle.

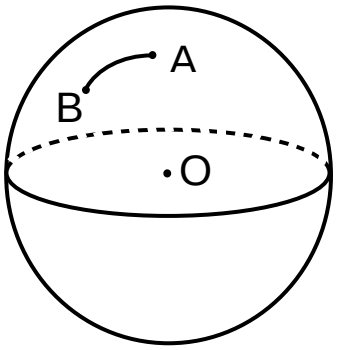
Finding Distance on a sphere

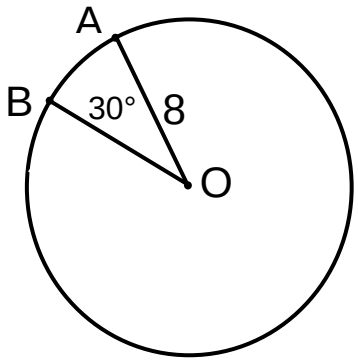


*Think of A and B as being on a great circle and find the length of the arc.

Notation: The distance from A to B is $d\overset{\frown}{AB}$. It is the same as the arc *length* (not the arc *measure*).

Example: Find $d\overset{\frown}{AB}$ if the radius of the sphere is 8 and $m\overset{\frown}{AB} = 30°$.





$d\overset{\frown}{AB} = \frac{30}{360}$ of the circumference of the great circle

$= \frac{1}{12}(2\pi r)$

$= \frac{1}{12}(2\pi)(8)$

$= \frac{4\pi}{3}$ units

Distance between 2 points on a sphere: The distance from point A to B on a sphere is given by

$$d\overset{\frown}{AB} = \frac{\theta}{360°}(2\pi r)$$

where θ is the measure of the arc in degrees, and r is the radius of the sphere.

Sample Problem: Find the length of each arc with the given angle measure on a sphere with radius 3 m.

1. $m\overset{\frown}{AB} = 15°$

   $d\overset{\frown}{AB} = \frac{15}{360}(2\pi)(3) = \frac{90\pi}{360} = \frac{\pi}{4}$

2. $m\overset{\frown}{AB} = 45°$

   $d\overset{\frown}{AB} = \frac{45}{360}(2\pi)(3) = \frac{270\pi}{360} = \frac{3\pi}{4}$

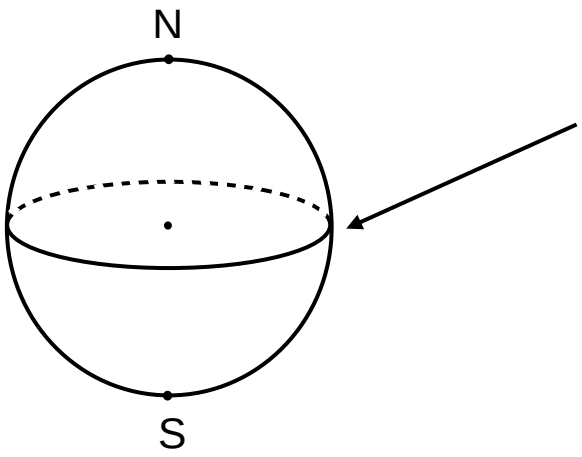
Sample Problem: If the radius of the earth is taken as 3950 miles and 2 cities are 9° apart, how far is a plane flight between cities?

$d\overset{\frown}{AB} = \frac{9}{360}(2\pi)(3950) = \frac{395\pi}{2} \approx 620$ miles

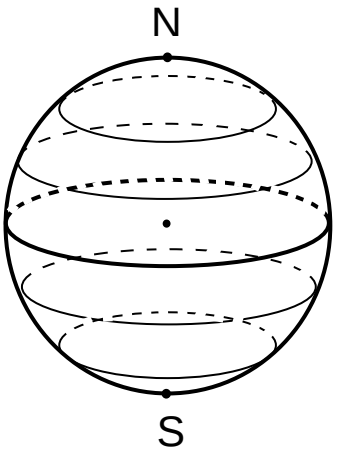
Sample Problem: What figure represents the intersection of 2 distinct spheres

1. If they are tangent?	a point
2. If they are disjoint?	empty set
3. If they are not tangent or disjoint?	a circle

Latitude:



The equator is the great circle that divides the earth into the Northern and Southern hemispheres.

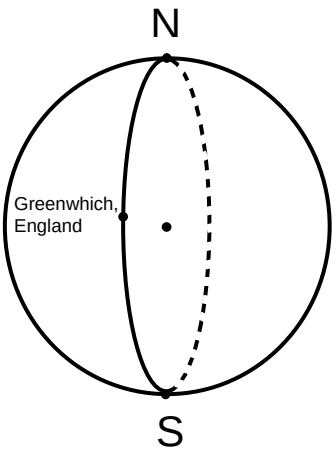


Planes that are parallel to the plane of the equator cut off smaller circles that appear parallel to the equator. these are called latitudinal lines or lines of latitude.

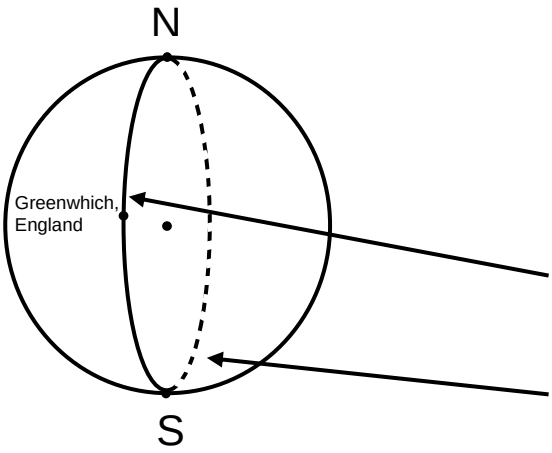
The equator is considered 0° and the poles are 90°.

Behind the degree measure we put N or S to show whether it is north of the equator or south of it.

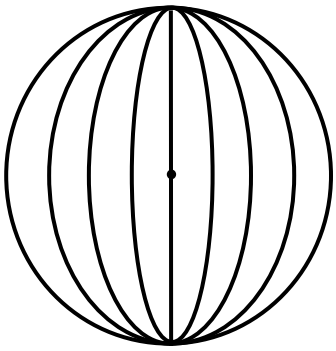
Longitude:



The great circle that passes through the North and South Poles and Greenwich, England, divides the earth into the Western and Eastern Hemispheres.



The semicircle through Greenwich is called the Prime Meridian. The other semicircle is called the International Date Line.



Great circles that pass through the poles are called longitudinal lines or lines of longitude.

Longitude is measured from the Prime Meridian (which is 0°). The International Date Line is 180°

Behind the degree we put E or W to shoe which side of the Prime Meridian the location is on.

**Note: Latitude circles have different radii measures. Longitude circles are all great circles.

Sample Problems: Find the city located at:

1. 35°N, 140°E Tokyo

2. 33°N, 45°E Baghdad

3. 20°N, 155°W Honolulu

4. 34°S, 151°E Sydney

5. 23°S, 43°W Rio de Janeiro

Sample Problems: Identify the location and give the latitude and longitude.

1. The highest mountain in the world, Mt. Everest, Nepal.

   28°N, 87°E

2. The tallest waterfall in the world, Angel Falls, Venezuela (near Esmeraldo, Venezuela).

   6°N, 62°W

3. The most violent volcanic eruption in modern history, Krakatau, Indonesia (near Jakarta).

   6°S, 105°E

Special Circles on the Globe

The Arctic Circle is at 66 ½° N.
The Antarctic Circle is at 66 ½° S.

The earth is tilted on its axis with respect to the sun at 23 ½°.

The Arctic and Antarctic circles mark the farthest distance from the poles with 24-hour darkness or light.

The Tropic of Cancer is 23 ½°N.
The Tropic of Capricorn is 23 ½°S.

Because of the earth's tilt, the Tropics of Cancer and Capricorn are the northernmost and southernmost latitudes reached by perpendicular rays from the overhead sun.

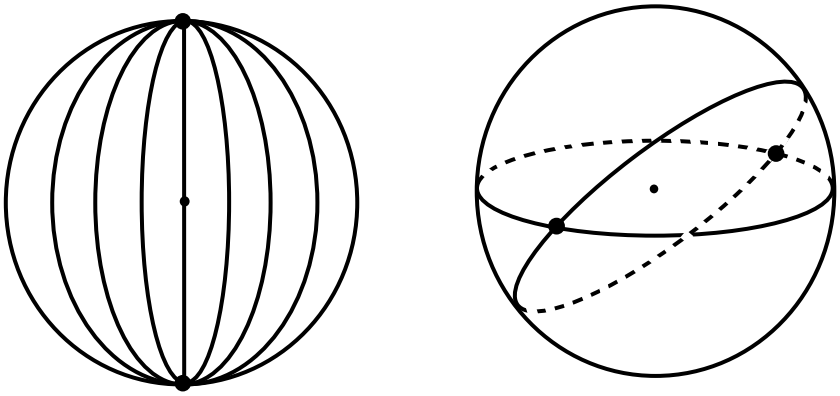
Riemannian Geometry (Spherical Geometry):

- Great circles are the “lines.”
- Points on the surface of a sphere are the “points.”
- The surface of the sphere is the “plane.”

Questions:

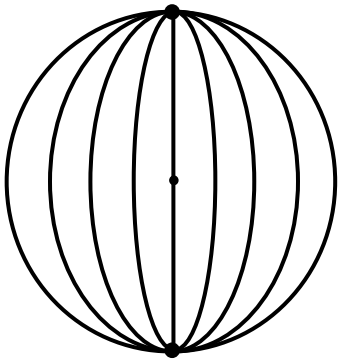
1. Do intersecting lines intersect at exactly one point?

   no, they intersect at 2 points

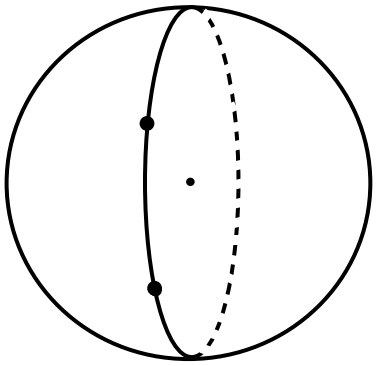


2. Do any two points determine exactly one line?

   - If the points are the ends of diameters (antipodes), then they determine infinitely many lines.



- If the points are not the ends of diameters, then yes, they determining exactly one line.

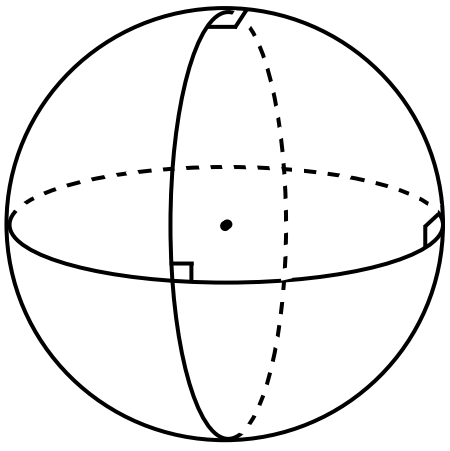


3. Through a point not on a line, how many parallel lines exist?

   none, all lines are great circles, and all great circles intersect in two points.

4. How many degrees are in an equilateral right triangle?

   3 x 90° = 270°



Chapter 10 Vocabulary:

- closed surface
- cube
- diagonal
- dihegral angle
- edge
- equator
- face
- great circle
- half-space
- hemisphere
- horizon
- interior of a dihedral angle
- international date line
- latitude
- line parallel to a plane
- line perpendicular to a plane
- longitude
- measure of a dihedral angle
- non-Euclidean geometry
- opposite edges of a parallelepiped
- opposite faces of a parallelepiped
- parallelepiped
- perpendicular bisecting plane
- perpendicular lines in space
- perpendicular planes
- perspective drawing
- plane angle
- prime meridian
- Riemannian geometry
- secant plane
- solid
- spherical geometry
- tangent
- vanishing point